Financial Calculations

The value of money

Annual Equivalent Rate (AER)

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

gives the value of the AER, r, given a nominal interest rate, i,

where n is the number of compounding periods per year

Example

A building society offers 6% gross interest on a savings account. Interest is paid into this account every 3 months. Calculate the AER.

Substituting $i = 0.06$, and $n = 4$ into $r = \left(1 + \frac{i}{n}\right)^n - 1$ gives $r = \left(1 + \frac{0.06}{4}\right)^4 - 1 = 1.015^4 - 1$

So $r = 1.06136 - 1 = 0.06136$ The **AER = 6.14%** (to 3sf)

Present and future value of money

$$FV = PV(1+r)^n \qquad PV = \frac{FV}{(1+r)^n}$$

where FV is the future value and PV the present value of a sum of money,
r is the interest rate expressed as a decimal and n is the number of time periods

Example

A sum of £15 000 is invested in an account with a fixed annual equivalent rate of 5.80% per annum. Calculate the amount of money in the account at the end of six years.

Substituting $PV = 15\,000$, $r = 0.058$ and $n = 6$ into $FV = PV(1+r)^n$

gives $FV = 15\,000(1+0.058)^6 = 15\,000 \times 1.058^6 =$ **£21 038.04** (to the nearest p)

Example

Find the sum of money you would need to invest to give £20 000 after ten years in an account with a fixed annual equivalent rate of 6.25% per annum.

Substituting $FV = 20\,000$, $r = 0.0625$ and $n = 10$ into $PV = \dfrac{FV}{(1+r)^n}$

gives $PV = \dfrac{20\,000}{(1+0.0625)^{10}} = \dfrac{20\,000}{1.0625^{10}} =$ **£10 907.89** (to the nearest p)

When the rate is fixed, **recurrence relations** can be used to find the value of an investment or the balance of an account at the end of each month or year.

Example

A student pays for a computer costing £399 with a credit card which charges 1.2% interest each month. She pays back £60 each month until the balance is less than £50, then makes one final payment to settle the account. Find the total amount paid and the interest paid as a percentage of the original price.

n	*Bn*
0	399.00
1	343.79
2	287.91
3	231.37
4	174.14
5	116.23
6	57.63

(a) The recurrence relation that gives the remaining balance each month is

$$B_{n+1} = 1.012B_n - 60$$

A graphic calculator can be used to complete a table to give the balance at the end of each month.

The student makes 6 payments of £60, then a final payment of £57.63.
The total amount paid = 6 × £60 + £57.63 = **£417.63**

(b) The interest as a % of the original price = $\dfrac{18.63}{399} \times 100$ = **4.67%** (to 3sf)

Annual Percentage Rate (APR)

$$C = \sum_{k=1}^{m} \left(\frac{A_k}{(1+i)^{(t_k)}} \right)$$

where i is the APR expressed as a decimal,
k is the number identifying a particular instalment,
A_k is the amount of the instalment k,
t_k is the interval in years between the payment of the instalment and the start of the loan.

Example

A borrower is lent £8000 on 1st May 2008 and agrees to repay this loan by a single repayment of £9000 on 31st May 2009. Find the APR.

For 1 repayment, the APR, i, can be found as a decimal from $\boldsymbol{C = \dfrac{A}{(1+i)^n}}$

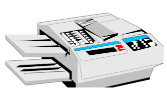
where C = 8000, A = 9000 and n = 1 + 30 ÷ 365 = 1.082 192 years.

$$8000 = \frac{9000}{(1+i)^{1.082\,192}} \quad \Rightarrow \quad (1+i)^{1.082\,192} = \frac{9000}{8000}$$

$$\Rightarrow \quad (1+i)^{1.082\,192} = 1.125 \quad \Rightarrow \quad 1+i = 1.125^{\frac{1}{1.082\,192}}$$

$$\Rightarrow \quad 1+i = 1.114\,981 \quad \Rightarrow \quad i = 0.114\,981$$

The APR is 11.5% (to 3sf)



Example
A loan of £7000 is repaid in annual instalments of £1000, £2000, £3000 and £4000.
(a) Show that the APR lies between 12.6% and 13%
(b) Use the interval bisection method to find the APR correct to 1 decimal place.

(a) When $i = 0.126$, $$C = \frac{1000}{1.126} + \frac{2000}{1.126^2} + \frac{3000}{1.126^3} + \frac{4000}{1.126^4} = 7055.25$$

When $i = 0.13$, $$C = \frac{1000}{1.13} + \frac{2000}{1.13^2} + \frac{3000}{1.13^3} + \frac{4000}{1.13^4} = 6983.67$$

As the true value of C, £7000 lies between these values, then the true value of i must lie between 0.126 and 0.13. So the APR lies between 12.6% and 13%

(b) **1st bisection** The mid-point of the interval $0.126 < i < 0.13$ is 0.128

When $i = 0.128$, $$C = \frac{1000}{1.128} + \frac{2000}{1.128^2} + \frac{3000}{1.128^3} + \frac{4000}{1.128^4} = 7019.33$$

This is too high, indicating that 0.128 is too low, so i must lie between 0.128 and 0.13.

2nd bisection The mid-point of the interval $0.128 < i < 0.13$ is 0.129

When $i = 0.129$, $$C = \frac{1000}{1.129} + \frac{2000}{1.129^2} + \frac{3000}{1.129^3} + \frac{4000}{1.129^4} = 7001.47$$

This is too high, indicating that 0.129 is too low, so i must lie between 0.129 and 0.13.

3rd bisection The mid-point of the interval $0.129 < i < 0.13$ is 0.1295

When $i = 0.1295$, $$C = \frac{1000}{1.1295} + \frac{2000}{1.1295^2} + \frac{3000}{1.1295^3} + \frac{4000}{1.1295^4} = 6992.56$$

This is too low, indicating that 0.1295 is too high, so i must lie between 0.129 and 0.1295. This means that the APR lies between 12.9% and 12.95%

The **APR = 12.9% correct to 1 decimal place**.

Indices

Percentage Change

$$\%\text{ change} = \frac{\text{current index} - \text{previous index}}{\text{previous index}} \times 100$$

Example

The table below gives the consumer price indices for Food (including non-alcoholic beverages) and Clothing (including footwear) at three monthly intervals.

(2005 = 100)	**January 2007**	**April 2007**	**July 2007**	**October 2007**	**January 2008**
Food	104.4	106.2	105.5	109.1	110.8
Clothing	92.0	93.7	89.8	92.5	87.5

(a) Calculate the percentage change between January 2007 and January 2008 in the price of: (i) Food (ii) Clothing.

(b) A shopper's average weekly food bill was £75.40 in July 2007.
 (i) Estimate the cost of the same food in January 2008.
 (ii) Give one possible reason for the change in cost.

(a) (i) $\%\text{ change in the price of food} = \frac{\text{Jan 2008 index} - \text{Jan 07 index}}{\text{Jan 07 index}} \times 100$

$$= \frac{110.8 - 104.4}{104.4} \times 100 = \mathbf{6.13\%} \text{ (to 3sf)}$$

(ii) $\%\text{ change in the price of clothing} = \frac{\text{Jan 2008 index} - \text{Jan 07 index}}{\text{Jan 07 index}} \times 100$

$$= \frac{87.5 - 92.0}{92.0} \times 100 = \mathbf{-\,4.89\%} \text{ (to 3sf)}$$

(b) (i) Estimate of cost in January 2008 $= \frac{£75.40}{105.5} \times 110.8 = \mathbf{£79.19}$

(ii) The seasonal cost of some foods (eg fruit and vegetables).

Weighted Average

The effective cost of a commodity $= \sum_i w_i\, p_i$

where w_i represents the proportion of the commodity bought at a price p_i at outlet i

Example

52% of the sales of cauliflowers in a small town are from a supermarket for 85 pence each. 34% are from a greengrocer's at 80 pence each and the rest from a farm at 60 pence each. Calculate the effective cost of a cauliflower in this town.

Effective cost = 0.52 × 85 + 0.34 × 80 + 0.14 × 60 = **79.8 pence**

Laspeyres index formula: $I_L = \left(\dfrac{\sum P_{it} Q_{i0}}{\sum P_{i0} Q_{i0}}\right) \times 100$

Paasche index formula: $I_P = \left(\dfrac{\sum P_{it} Q_{it}}{\sum P_{i0} Q_{it}}\right) \times 100$

Fischer index formula: $I_F = \sqrt{I_L \times I_P}$

where

P_{i0} is the price of commodity I at time 0, Q_{i0} is the quantity of commodity I at time 0

P_{it} is the price of commodity I at time t and Q_{it} is the quantity of commodity I at time t

Example

The table gives the prices and quantities sold of two types of tennis rackets, Ace and Excel, manufactured by a company.

	January (J)		**February (F)**		**March (M)**		**April (A)**	
	Price	**Quantity**	**Price**	**Quantity**	**Price**	**Quantity**	**Price**	**Quantity**
Ace	£55	1200	£56	1100	£58	1500	£59	1800
Excel	£29	2500	£30	2800	£32	2500	£39	5400

(a) Calculate a fixed-base Laspeyres index for the data from January to April.
(b) Calculate a fixed-base Paasche index for the data from January to April.
(c) Use your answers to (a) and (b) to find a Fischer index for the data from January to April.

(a) $I_L = \left(\dfrac{\sum P_{iA} Q_{iJ}}{\sum P_{iJ} Q_{iJ}}\right) \times 100 = \dfrac{59 \times 1200 + 39 \times 2500}{55 \times 1200 + 29 \times 2500} \times 100 = 121.5162 = \mathbf{121.52}$ (to 2dp)

(b) $I_P = \left(\dfrac{\sum P_{iA} Q_{iA}}{\sum P_{iJ} Q_{iA}}\right) \times 100 = \dfrac{59 \times 1800 + 39 \times 5400}{55 \times 1800 + 29 \times 5400} \times 100 = 123.9437 = \mathbf{123.94}$ (to 2dp)

(c) $I_F = \sqrt{I_L \times I_P} = \sqrt{121.5162 \times 123.9437} = \mathbf{122.72}$ **(to 2dp)**

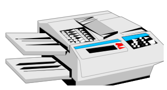
Example

Given that the Laspeyres index from year 0 to year 1 is 98.32, from year 1 to year 2 is 105.17, and from year 2 to year 3 is 106.45, calculate a Laspeyres chain index from year 0 to year 3.

Laspeyres chain index from year 0 to year 3, $I_{03} = \dfrac{I_{01}}{100} \times \dfrac{I_{12}}{100} \times I_{23}$

$= 0.9832 \times 1.0517 \times 106.45$

$= \mathbf{110.07}$ (to 2dp)



Teacher Notes

Unit Advanced Level, Mathematical Principles for Personal Finance

Notes on Activity

Pages 1 to 5 give examples of the types of calculations that candidates are likely to meet in the FSMQ examination. It is suggested that learners work through these as part of their revision. Ideally they should try each question for themselves, covering up the given solution until they are ready to check the answer they have found.

Learners also need to be aware that they will be asked questions based on the tables or diagrams that are included in the Data Sheet that they will receive during the last two weeks before the examination.